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TEMPORAL ANALYSIS OF EARTHQUAKE AND QUANTILE FUNCTION ESTIMATION

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Abstract. *This study explores a novel method for estimating and approximating quantile and quantile density functions based on known moments. In this context, the moments are represented by time intervals (waiting times) between earthquakes recorded in seismic catalogs. The research utilizes global seismic data on earthquake waiting times and evaluates three different approximation models: frequency moments, conventional moments, and transposition moments. While multiple approaches exist for estimating quantile functions, the key advantage of our method lies in its reliance solely on moment information.*

Key Words: *Earthquake, waiting time, quantile function*

Introduction

Among the notable outcomes of previous research are well-established insights into the temporal characteristics of seismicity, which align with contemporary perspectives on the fractal nature of tectonic structures, fault systems, and hypocenter distributions. These findings support the understanding of seismic processes as inherently complex and dynamic, where earthquake occurrences exhibit "switching" or "shifting" behavior – alternating between phases of heightened and diminished seismic activity. Despite this progress, many core aspects of how time intervals between earthquakes are distributed remain insufficiently understood, and in some cases, have yet to be thoroughly investigated [1–3].

Currently, the moment problem holds a significant place in statistics and finds applications in fields such as mathematics, financial mathematics, economics, and insurance. Over the past three centuries, it has been extensively explored in numerous publications, yet it remains a subject of great mathematical interest to this day [4–6].

In this study, we focus on the approximation and estimation of the quantile function, assuming that the moments are known [7].

We can say that the moment problem has the only solution when the system of equations $\int x^j dF(x) = \int x^j dG(x); j = 0,1,..$, has one solution, *F*=*G*.

Methods

There exist various nonparametric approaches for estimating the quantile function. For instance, Harrell and Davis [8] examine the use of statistical order statistics, while Bolancé et al. [9] and Brewer [10] explore quantile estimation based on Bernstein polynomials.

The innovative aspect of this research lies in its applicability in cases where limited information about the underlying distribution function is available – specifically, when only the moments are known. This advantage allows for more flexible modeling in data-sparse scenarios.

In this study, we aim to combine the analysis of time intervals (waiting times) between earthquakes with the approximation of results using the quantile function, supported by modern computational techniques. Our approach incorporates both classical linear and nonlinear methods for studying time distribution patterns.

By analyzing waiting time series originating from various sources, we seek to uncover key characteristics in the temporal dynamics of seismic activity. This project aligns closely with fundamental challenges in Earth sciences and involves the integration of advanced data analysis techniques within a custom-developed software platform.

For the estimation, we consider three distinct models. The first utilizes frequency moments, the second relies on conventional moments, and the third is based on transposition moments.

Results and discussion.

For the analysis of time intervals (waiting times), data from the Southern California Seismic Catalog were used. Specifically, we utilized records from the Southern California Local Earthquake Catalog, accessible at http://www.data.scec.org/ftp/catalogs/. The dataset spans the period from 1932 to 2013.

This catalog is considered highly reliable due to its near-continuous data collection throughout the entire period.

Subsequently, using software developed at the M. Nodia Institute of Geophysics, Ivane Javakhishvili Tbilisi State University, we processed the dataset to extract waiting times between earthquake events.

Based on these waiting time sequences, we computed an approximate estimate of the quantile function using the first model. This result will be compared with the well-known Harrell-Davis estimator. The analysis employs the following formulas:

$$\widehat{Q_{HD}} = \sum_{i=1}^{n} X_{(i)} \int_{\frac{i-1}{n}}^{\frac{i}{n}} \beta(y, \lfloor \alpha x \rfloor + 1, \alpha - \lfloor \alpha x \rfloor + 1) dy$$

$$\widehat{Q_{\alpha}^{-}}(x) = \sum_{i=1}^{n+1} \Delta X_{(i)} B_{\alpha}\left(\frac{i-1}{n}, x\right) = \sum_{i=1}^{n} \Delta X_{(i)} [B_{\alpha}\left(\frac{i-1}{n}, x\right) - B_{\alpha}\left(\frac{i}{n}, x\right)]$$

The behavior for different parameters for first model will be as follows (Fig.1- Fig.3).

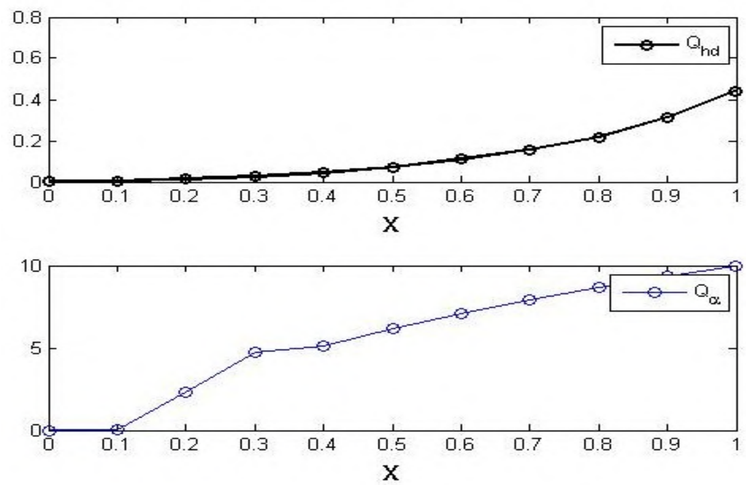

Fig. 1. Comparison of estimates of Harrell Davis and the first model (frequency moments) for the waiting time when. $\alpha = 20$, $n=100$

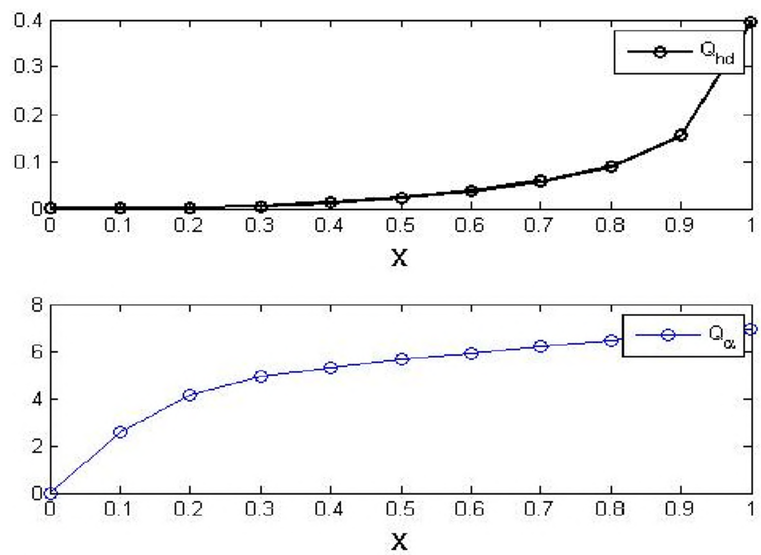

Fig. 2. Comparison of the estimates of Harrell Davis and the first model (frequency moments), for the waiting time when $\alpha = 50$, $n=100$

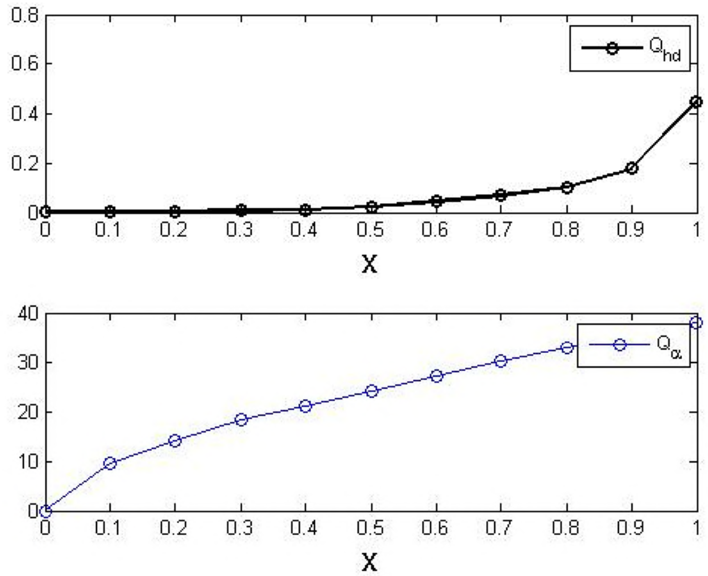

Fig. 3. Comparison of the estimates of Harrell Davis and the first model (frequency moments), for the waiting time when $\alpha=100$, $n=100$

The results of the research will help to understand the nature of seismic processes and what most important it will contribute in the future to solving the problem of understanding the nature of strong earthquakes.

Conclusion

Our study focuses on analyzing the time intervals between earthquake events and employs specific datasets to approximate and estimate the quantile function. This approach has broad applicability in fields such as financial mathematics, economics, and insurance. The project takes on a global scope, addressing significant challenges across multiple scientific disciplines. It holds both theoretical and practical value, as understanding the temporal distribution of earthquakes is essential for assessing seismic hazards and making meaningful advancements in comprehending the underlying mechanisms of earthquakes.

References

1. Telesca L., Matcharashvili T., Chelidze T., Investigation of the temporal fluctuations of the 1960–2010 seismicity of Caucasus. // Nat. Hazards Earth Syst. Sci., Göttingen Germany, 12, 2012, pp. 1905–1909.
2. Talbi A., Yamazaki F., Sensitivity analysis of the parameters of earthquake recurrence time power law scaling. // J. Seismol, Springer, Netherlands, 13, 2009, pp. 53−72.
3. Molchan G., Interevent time distribution in seismicity: a theoretical approach. // Pure Appl Geophys., Springer, Netherlands, 162, 2005, pp. 1135−1150.
4. Mnatsakanov R., Sborshchikovi A., Recovery of a Quantile Function from Moments. // Journal of Computational and Applied Mathematics, Elsevier, Netherlands, 315, 2017, pp. 354-364.
5. Mnatsakanov R., Sborshchikovi A., Recovery of quantile and quantile density function using the frequency moments. // Journal of Statistics and Probability Letters, Elsevier, Netherlands, 140, 2018, pp. 53-62.
6. Sborshchikovi A., On Nonparametric Quantile Function Estimation Using Transformed Moments. // Bulletin of the Georgian National Academy of Sciences, Tbilisi, Georgia, 11(3), 2017, pp. 22-27.
7. Cheng C., Parzen E., Unified estimators of smooth quantile and quantile density functions. // J. Statist. Plann. Inference, Elsevier, Netherlands, 59, 1997, pp. 291–307.
8. Harrell F.E., Davis C.E., A new distribution-free quantile estimator. // Biometrika, Oxford University Press, 69, 1982, pp. 635–640.
9. Bolance C., Guillen M., Nielsen, J., Transformation kernel estimation of insurance claim cost distributions. in: M. Corazza, et al. (Eds.), Mathematical and Statistical Methods for Actuarial Sciences and Finance, Springer-Verlag, Italy, 2010, pp. 43–51.
10. Brewer K.R.W., Likelihood Based Estimation of Quantiles and Density Estimation, 1986.